Relativistic channeling by intense laser pulse in overdense plasmas

Baifei Shen[1,2] and M. Y. Yu[1]

[1]*Institüt für Theoretische Physik I, Ruhr-Universität Bochum, D-44780 Bochum, Germany*
[2]*Shanghai Institute of Optics and Fine Mechanics, P.O. Box 800-211, Shanghai 201800, China*
(Received 10 January 2003; revised manuscript received 6 May 2003; published 29 August 2003)

Channeling in overdense plasma by relativistic laser pulse is investigated. The critical laser power needed to maintain a plasma channel as well as the mode profiles of the electromagnetic fields in the channel cross section are obtained analytically. A scaling law showing that the critical power is proportional to the square of the background plasma density is found.

In many practical applications, laser pulses are required to propagate long distances with preferably little or no attenuation. Plasma channels are ideal for this purpose. A number of approaches for guiding intense radiation in plasma channels have been proposed. For example, low-intensity laser pulses can be guided by hollow capillaries [1,2] or preformed plasma channels [3,4], and high-intensity pulses by relativistic channeling in underdense plasmas or neutral gases [5–16]. In the latter case, stable channeling can occur if the laser power P exceeds the critical value P_{cr} for relativistic self-focusing [5], where $P_{cr} = 17\omega_L^2/\omega_{pe}^2$ GW, ω_L and ω_{pe} are the laser and plasma frequencies, respectively. If the power is still larger, electrons can be completely expelled from the channel, which is maintained by a balance between the relativistic ponderomotive force and the space-charge electric field generated by the displaced electrons piled up at its edge. Inside the channel the laser pulse can propagate like in a waveguide with little attenuation. In overdense plasma, a laser pulse with normalized vector potential $a \equiv eA/mc^2 > 1$ can also propagate, and still higher power can lead to self-focusing and self-channeling. Although essential for the fast-igniter scheme of inertial fusion [17,18], laser channeling in overdense plasma is still not yet well understood compared to channeling in underdense plasmas [19–23]. In this paper, we investigate self-consistent stationary states of laser-formed channels in overdense plasmas. Although a real laser pulse usually consists of many modes, stationary channel states containing only one or a few modes are possible when the laser pulse propagates into preformed (say, by the preceding pulses) channels because the modes propagate at different speeds and therefore tend to separate from each other. Several types of stationary channels are studied, distinguished by the presence or absence of electrons and ions inside the channel, as determined by the time scale and laser power at which the channel is maintained. A simple scaling law for the critical power for channeling, as well as the structure of the transverse field, is obtained. It is shown that, in contrast to that in underdense plasmas, at high plasma densities the critical laser power for channeling scales as the square of the density.

The equations describing the relativistic interaction of intense laser light with cold electron fluid in a background of immobile ions are

$$\nabla^2 \mathbf{A} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{A} = \frac{1}{c}\frac{\partial}{\partial t}\nabla\varphi + \frac{4\pi e}{c}n_e\mathbf{u}, \qquad (1)$$

$$\nabla^2 \varphi = 4\pi e(n_e - n_i), \qquad (2)$$

$$m\gamma\mathbf{u} = \frac{e}{c}\mathbf{A} + \nabla\psi, \qquad (3)$$

$$\frac{\partial\psi}{\partial t} = e\varphi - mc^2(\gamma - 1), \qquad (4)$$

where $\gamma = (1 - u^2/c^2)^{-1/2}$ is the relativistic factor, n_e and n_i are the electron and ion densities, respectively, ψ is the canonical momentum in the direction of laser propagation. The Coulomb gauge is used.

We are interested in the cross-sectional structure of the laser field in the channel. In the absence of energy loss [24], the vector potential of the fundamental frequency can be written as $\mathbf{A} = A_0(r)e^{-in\theta}e^{i(\omega_L t - kz)}(\hat{e}_x + i\hat{e}_y)$, where n is the azimuthal wave number and θ the angle, respectively. Thus, there is no wave electric field in the direction of laser propagation, and we are considering transverse electric (TE) modes in the channel. For simplicity, only low-order modes will be considered. In reality, the incident laser pulse usually contains both TE and transverse magnetic (TM) modes, so that waves of different modes can coexist inside the channel. Only when the pulse is specially prepared, waves of a single mode may be realized in the channel.

For simplicity, we look for stationary states occurring when the ponderomotive force of the laser pulse is balanced by the electrostatic force arising from the charge separation caused by the electron displacement. That is, the evolution of the laser and the channel formation process are not considered here. Accordingly, we have

$$\frac{\partial\gamma}{\partial r} = \frac{\partial\phi}{\partial r}, \qquad (5)$$

where $\gamma = \sqrt{1 + a_0^2}$, $a_0 = eA_0/mc^2$, $\phi = e\varphi/mc^2$, and $r = r\omega_L/c$. Because the laser is circularly polarized, the relativistic factor γ is only a function of the radius r. The wave equation then becomes

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial a_0}{\partial r}\right) - \frac{a_0}{\gamma^2}\left(\frac{\partial a_0}{\partial r}\right)^2 - \frac{\gamma^2 a_0 n^2}{r^2}$$
$$- a_0\gamma N_i + (1 - k^2)a_0\gamma^2 = 0, \qquad (6)$$

where $k=kc/\omega_L$, $N_i=n_i/n_c$, and $n_c=m\omega_L^2/4\pi e^2$ is the critical density. The electron density $N_e=n_e/n_c$ is given by

$$N_e=\gamma^2 N_i+\frac{1}{\gamma}\left(\frac{\partial a_0}{\partial r}\right)^2+\frac{\gamma a_0^2 n^2}{r^2}-(1-k^2)a_0^2\gamma. \quad (7)$$

Inside the channel the laser pulse moves slower than that in vacuum, or $k<1$. In contrast to the case of underdense plasma, k can be very small in overdense plasma channels. It can even approach zero. Wave attenuation along the channel can be represented by a complex k.

For sufficiently large laser power and time scale, electron and even ion cavitation in the channel can occur. Since we are considering stationary states, it is convenient to define the boundaries R and R_0 (with $R>R_0$) for the electrons and ions, respectively. Thus, R_0 can be taken to be the radius of a preformed channel, and $R=0$ corresponds to the case of stationary ions. Inside an electron-free channel, Eq. (6) reduces to

$$\frac{\partial^2 a_0}{\partial r^2}+\frac{1}{r}\frac{\partial a_0}{\partial r}-\frac{n^2 a_0}{r^2}+(1-k^2)a_0=0, \quad (8)$$

and the solution is given by

$$a_0(r)=\alpha_n J_n(\sqrt{1-k^2}r), \quad (9)$$

which is the same as that for the electromagnetic wave fields inside a traditional waveguide. Balance between the electrostatic space-charge and ponderomotive forces at the boundary $r=R$ leads to

$$\left(\frac{\partial\gamma}{\partial r}\right)_R=-\frac{1}{2}N_i\frac{R^2-R_0^2}{R}, \quad R>R_0, \quad (10)$$

where R_0 is the radius of the vacuum (with no electron or ion) channel. If there is no preformed channel ($R_0=0$) and if the ions are unperturbed by the laser, Eq. (10) reduces to

$$\left(\frac{\partial\gamma}{\partial r}\right)_R=-\frac{1}{2}N_i R \quad (11)$$

for an electron-free channel containing stationary ions.

Axial quasistatic magnetic fields in plasma channels have been widely studied [23,25]. However, here the axial magnetic field associated with the fundamental frequency still needs to be considered. From Eq. (9), we obtain

$$B_\parallel=\alpha_n\sqrt{1-k^2}J_{n-1}e^{i[(-n+1)\theta+\pi/2]}e^{i(\omega_L t-kz)} \quad (12)$$

for the normalized axial channel magnetic field. For $n=1$, the axial magnetic field is of the form $J_0(\sqrt{1-k^2}r)$. This is the TE$_{01}$ mode. Using Eq. (10), we obtain
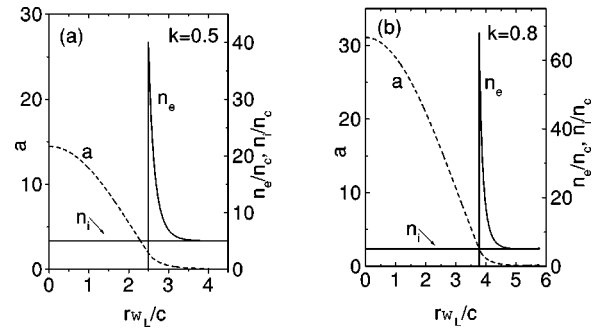

FIG. 1. Cross-sectional profile of the normalized vector potential a and electron density n_e for the TE$_{11}$ mode. The ambient plasma density is $n_i=5n_c$, for (a) $k=0.5$ and (b) $k=0.8$.

$$\alpha_1^2=\frac{(R^2-R_0^2)^2 N_i^2}{8(1-k^2)(RJ_0-J_1)^2}$$
$$\times\left[1+\sqrt{1+\frac{16(1-k^2)(RJ_0-J_1)^2}{(R^2-R_0^2)^2 N_i^2 J_1^2}}\right]. \quad (13)$$

For $n=0$, the axial magnetic field is of the form $J_1(\sqrt{1-k^2}r)$, corresponding to the TE$_{11}$ mode. In this case

$$\alpha_0^2=\frac{(R^2-R_0^2)^2 N_i^2}{8(1-k^2)R^2 J_1^2}\left[1+\sqrt{1+\frac{16R^2(1-k^2)J_1^2}{(R^2-R_0^2)^2 N_i^2 J_0^2}}\right]. \quad (14)$$

We begin the numerical calculation with the case of relatively low ambient density $N_i=5$. In Fig. 1, the laser field and plasma density profile for (a) $k=0.5$ and (b) $k=0.8$ are given. The electrons are expelled by the ponderomotive force and they piled up at the outside channel edge. Furthermore, larger k corresponds to larger laser power, larger channel diameter, and larger group velocity of the electromagnetic waves. In Figs. 2(a) and 2(b), the profiles corresponding to the TE$_{01}$ mode are given for the same two k values. We see that higher laser power is needed to propagate the TE$_{01}$ mode at the same group velocity as that of the TE$_{11}$ mode. Therefore, for the same laser power, electromagnetic waves in the TE$_{11}$ mode propagate faster than that in the TE$_{01}$ mode. In practice, the laser pulse has a nearly Gaussian profile. If the laser power is sufficiently large, both TE$_{11}$ and TE$_{01}$ modes will appear in the channel. However, since waves in the TE$_{11}$
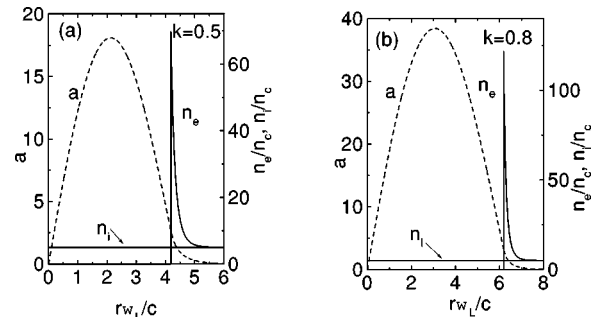

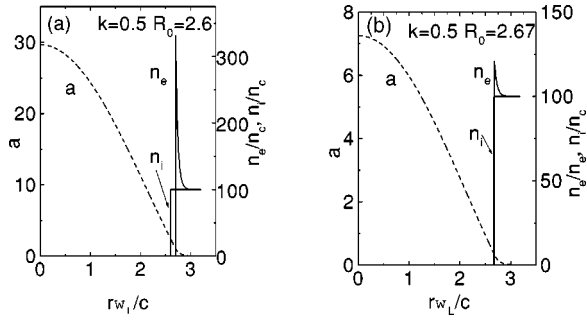FIG. 2. Same as in Fig. 1, but for the TE$_{01}$ mode.

FIG. 3. Cross-sectional profile of a and n_e for the TE_{11} mode with $k=0.5$ in preformed plasma channels. The ambient plasma density is $n_i=100n_c$, and the radii of the preformed vacuum channels are (a) $R_0=2.6$ and (b) $R_0=2.67$.

mode have larger group velocity, it will move to the pulse front, followed by the TE_{01}, and perhaps also other higher modes. Such behavior was in fact observed in three-dimensional particle-in-cell simulations of laser propagation in underdense plasmas [11].

For much higher ambient plasma density, say $n_i=n_e=100n_c$, it is rather difficult for a laser pulse of present-day power to propagate. However, a pulse can easily propagate in a preformed channel. Laser propagation in large preformed plasma guides, say, with 200 μm cross section, have been considered earlier [3,4]. Here we are interested in preformed channels with radius of the order of the laser wavelength. Figure 3 shows a narrow vacuum channel formed by a laser pulse with $k=0.5$ in an ambient plasma of uniform density $n_i=100$. For Fig. 3(a), the radius of the vacuum channel is $R_0=2.6$. We see that at the center of the channel the normalized vector potential a is about 29 for the TE_{11} mode. In Fig. 3(b), the channel is slightly larger ($R_0=2.67$). The vector potential at the center of the channel is reduced to about 7.3. Thus a much lower power is needed to maintain a larger channel. Figure 4 shows the limiting case of the TE_{01} mode at $k=0$. Here the group velocity of the laser field vanishes and there is only electric field in the transverse direction.

From the boundary conditions relating the laser fields in the vacuum and inside the channel, we have $a_L=(1/2)(1+k)a$ for the vector potential of incident laser pulse. Thus, for circular polarization the laser power is [26]
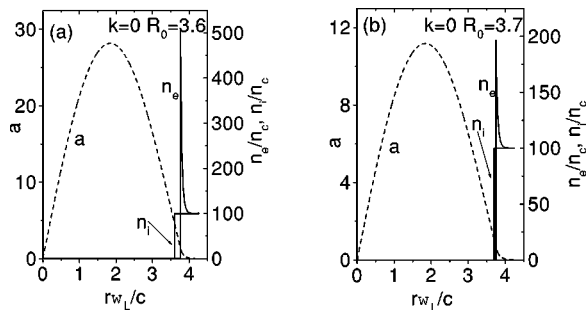

FIG. 4. Cross-sectional profile of a and n_e for the TE_{01} mode with $k=0$ in preformed plasma channels. The ambient plasma density is $n_i=100n_c$. The radii of the preformed vacuum channels are (a) $R_0=3.6$ and (b) $R_0=3.7$.
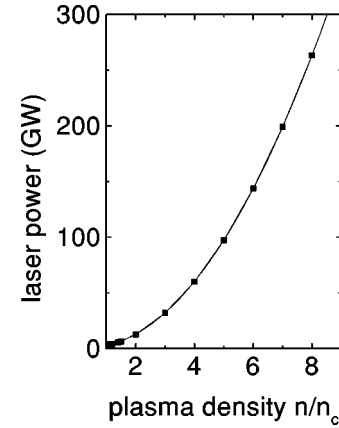

FIG. 5. Critical laser power needed to maintain a channel in overdense plasma as a function of the plasma density. The solid line is for $P_L=1.87-3.63(n/n_c)+4.54(n/n_c)^2$. That is, the laser power scales as n^2.

$$P_L=\frac{13.8}{\pi}\int_0^\infty a_L^2(r)r\,dr=\frac{13.8}{4\pi}\int_0^\infty (1+k)^2a^2(r)r\,dr,$$

(15)

which is in units of GW.

The lowest, or critical, laser power $P_L=P_{cr}$ needed to maintain a channel in an overdense plasma with unperturbed ions ($R_0=0$) is for a TE_{11} mode with $k=0$. This corresponds to a standing (in the axial direction) wave structure in a channel of zero radius (that is, the channel is void of electrons, but the ions are unperturbed). The corresponding power is given in Fig. 5 as a function of the plasma density. We find that the required power is to a very good approximation given by the simple scaling relation $P_{cr}=1.87-3.63(n/n_c)+4.54(n/n_c)^2$ GW, or that at high densities the power is proportional to the square of the ambient plasma density. This result differs considerably from that for channeling in underdense plasmas or neutral gases [5,7–16,27]. We see also that a plasma at critical density would be most suitable for relativistic channeling. Furthermore, except for plasmas of density near critical, channeling can occur even for $k=0$.

In this paper we have studied the stationary (on the electron timescale) states of channels formed by high intensity lasers in overdense plasmas. Here the electron-free channels are formed by a balance between the laser ponderomotive force and the space-charge electrostatic field produced by the expelled electrons (the ions remain stationary in the time scale of interest). The study differs from earlier works on laser boring, which involve the thermal or ponderomotive hydrodynamic shocklike compression of the plasma electrons and ions by longer-pulse lasers on the ion time scale [6,27]. A scaling law for the critical laser power for channel maintenance in overdense plasmas is obtained. Simulations [22] showed that nonthermal energetic electrons can play a role during the channel formation process in overdense plasmas and can thus affect the initial mode behavior. However, they do not seem to affect the later channel states. On the

other hand, if the laser power is much above critical, it is expected that filamentation instabilities will occur [12,18].

The present work can help to determine how deep the igniter laser pulse can penetrate the compressed high-density plasma in the fast igniter scheme for inertial fusion [17,18]. Reflection and scattering involving parametric instabilities, pulse and plasma deformation, etc., because of interaction of the ultraintense light with the outer underdense preplasma region can severely reduce the laser energy arriving at the compressed fuel. To avoid such loss and deflection, a guiding device [18] can be inserted to the fuel containing shell, so that the igniter pulse arrives directly at the high-density compressed fuel with minimum loss.

Besides of importance to inertial fusion, ultra-high-intensity laser channels in high-density plasmas can have many other applications where localized high-intensity electromagnetic fields are required. Since only laser light of sufficient power can self-channel, the process can also be used to cut the prepulse of a laser pulse. Channeling in overdense plasmas also provides an alternative to channeling in underdense plasmas in applications such as electron and ion acceleration, long-distance self-focusing, high-order harmonic and x-ray generation, etc. In other applications, one may also need both intense light and high electron density. For example, recently it has been suggested that specially guided electromagnetic fields can be made to reach the ultrahigh intensities needed to detect QED nonlinearities [28–30]. The configuration discussed here seems to be more readily realizable than that (where normal metal waveguide is used) proposed by Brodin *et al.* [29]. Finally, our exact analytical results on the stationary channel states can also serve as a guide for experiments and simulations of the laser channeling process in overdense plasmas.

---

[1] A. Butler, D.J. Spence, and S.M. Hooker, Phys. Rev. Lett. **89**, 185003 (2002).

[2] F. Dorchies *et al.*, Phys. Rev. Lett. **82**, 4655 (1999).

[3] C.G. Durfee and H.M. Milchberg, Phys. Rev. Lett. **71**, 2409 (1993).

[4] J. Fan, E. Parra, and H.M. Milchberg, Phys. Rev. Lett. **84**, 3085 (2000).

[5] C.E. Max, J. Arons, and A.B. Langdon, Phys. Rev. Lett. **33**, 209 (1974);

[6] See C. E. Max, Lawrence Livermore National Laboratory Report No. UCRL-53107 1981 (unpublished), and the references therein.

[7] G.Z. Sun, E. Ott, Y.C. Lee, and P. Guzdar, Phys. Fluids **30**, 526 (1987).

[8] A.B. Borisov *et al.*, Phys. Rev. Lett. **68**, 2309 (1992).

[9] X.L. Chen and R.N. Sudan, Phys. Fluids B **5**, 1336 (1993).

[10] P. Monot *et al.*, Phys. Rev. Lett. **4**, 2953 (1995).

[11] A. Pukhov and J. Meyer-ter-Vehn, Phys. Rev. Lett. **76**, 3975 (1996).

[12] A.B. Borisov *et al.*, J. Phys. B **32**, 3511 (1999).

[13] E. Esarey *et al.*, Phys. Rev. Lett. **84**, 3081 (2000).

[14] B. Hafizi *et al.*, Phys. Rev. E **62**, 4120 (2000).

[15] H. Yang *et al.*, Phys. Rev. E **65**, 016406 (2001).

[16] X. Yuan *et al.*, Phys. Scr. **66**, 381 (2002).

[17] M. Tabak *et al.*, Phys. Plasmas **1**, 1626 (1994).

[18] R. Kodama *et al.*, Nature (London) **412**, 798 (2001).

[19] S.C. Wilks *et al.*, Phys. Rev. Lett. **69**, 1383 (1992); S.C. Wilks, Phys. Fluids B **5**, 2603 (1993).

[20] R.N. Sudan, Phys. Rev. Lett. **70**, 3075 (1993).

[21] A. Pukhov and J. Meyer-ter-Vehn, Phys. Rev. Lett. **79**, 2686 (1997).

[22] Y. Sentoku *et al.*, Phys. Rev. E **65**, 046408 (2002).

[23] A. Kim *et al.*, Phys. Rev. Lett. **89**, 095003 (2002); Phys. Rev. E **65**, 036416 (2002).

[24] J.R. Davies and J.T. Mendonca, Phys. Rev. E **62**, 7168 (2000).

[25] Z.M. Sheng and J. Meyer-ter-Vehn, Phys. Rev. E **54**, 1833 (1996).

[26] E. C. Jordan, *Electromagnetic Waves and Radiating Systems* (Prentice-Hall, Englewood Cliffs, NJ, 1964).

[27] W. Yu, M.Y. Yu, J. Zhang, and Z. Xu, Phys. Rev. E **58**, 6553 (1998).

[28] M. Soljacic and M. Segev, Phys. Rev. A **62**, 043817 (1997).

[29] G. Brodin, M. Marklund, and L. Stenflo, Phys. Rev. Lett. **87**, 171801 (2001).

[30] B. Shen and M.Y. Yu, Phys. Rev. Lett. **89**, 275004 (2002).